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State Splitting in Continuous Time STN-models

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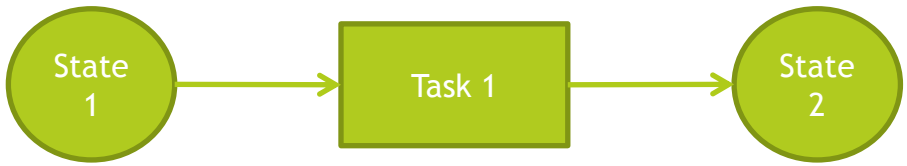
State Task Network models

- STN models
  - Are used for modeling batch process in both discrete and continuous time
- An STN graph consists of
  - Task nodes described as rectangles
  - State nodes described as circles
  - Directed arches
- States represent commodities
- Tasks transform one or more states into a new state
  - Tasks are preformed on units
- Arches describe batches of commodities moving through the graph

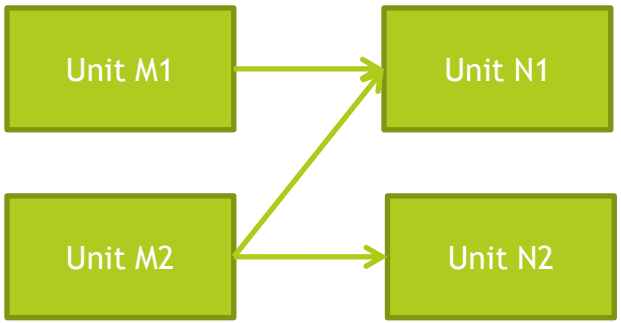
Example: A simple STN graph



Limited equipment connectivity

- Limited equipment connectivity
  - When at least one unit in a production stage is not connected to all units in the next stage
  - Common in many industries

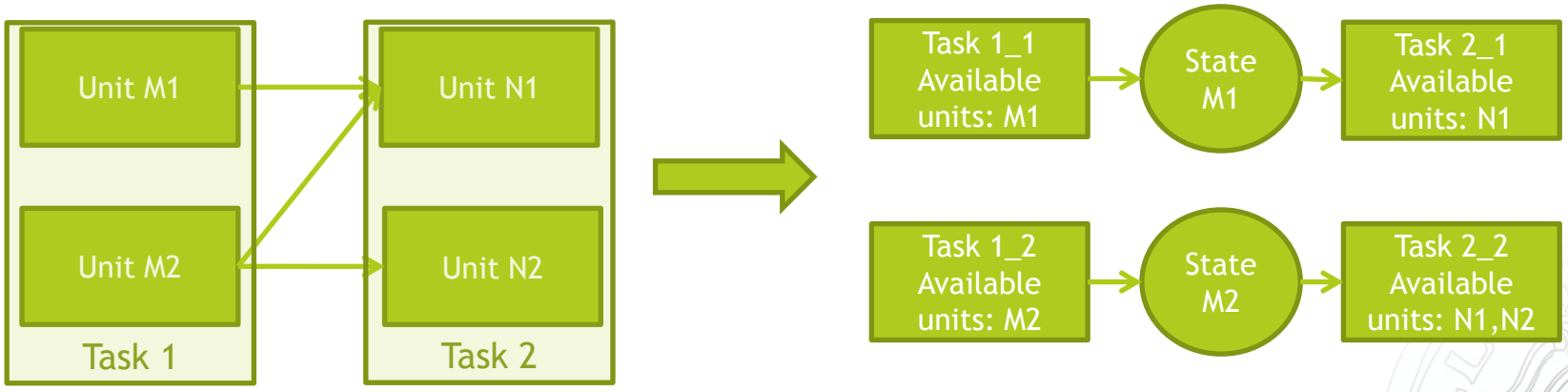
Example: Limited equipment connectivity



Task Splitting

- Kondili et al. proposed task splitting for including limited equipment connectivity in STN models
- Task Splitting does the following:
  - Duplicates tasks for units in the later stage to include limited connectivity
  - Task Splitting does not require any modifications to the mathematical model, only additional tasks are required
- The method has two drawbacks:
  - Increases the number of binary variables
  - Prohibits merging of batches

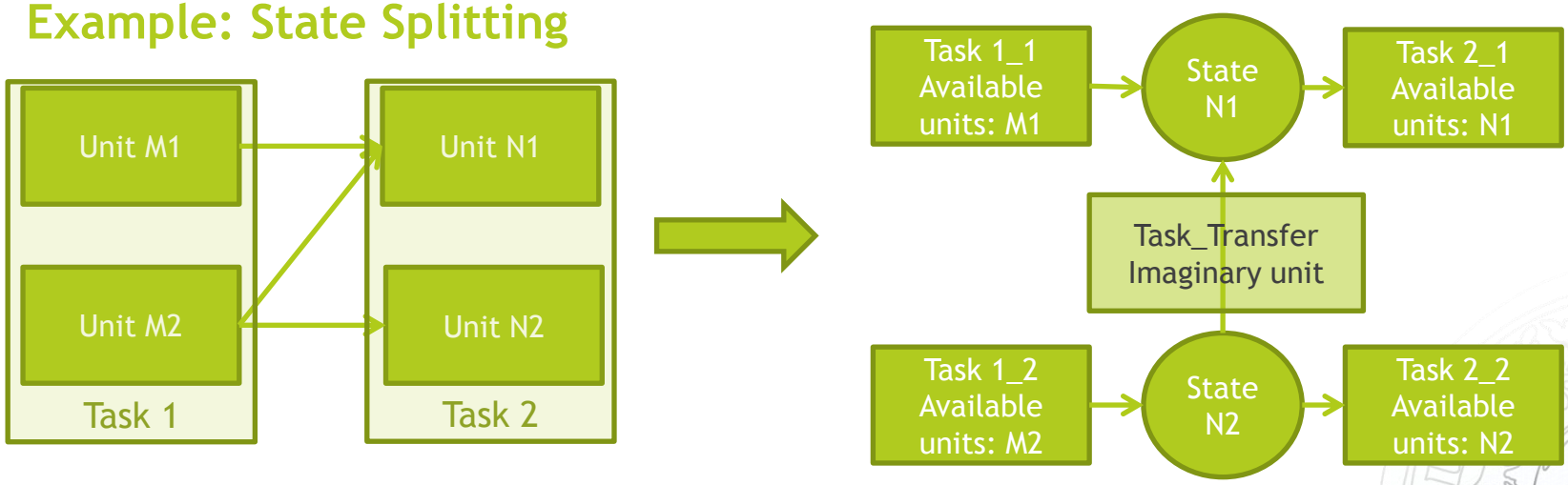
Example: Task Splitting



State splitting

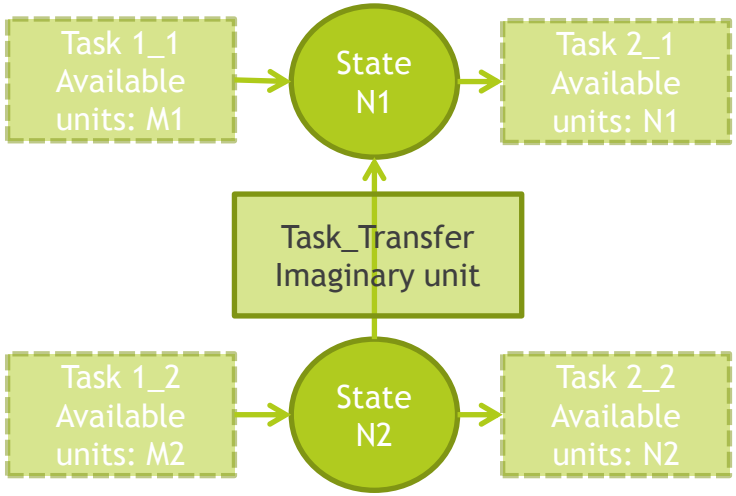
- Includes limited connectivity by splitting states and connecting them instead of splitting tasks
- Requires:
  - A new set of “imaginary” transfer tasks and units
  - Inclusion of the new task(s) into the material balance constraint
  - Reformulated batch size constraint for transfer tasks

Example: State Splitting



Imaginary transfer unit and task

- For every unit configuration with limited connectivity State Splitting adds a imaginary unit and task connecting two States
  - The transfer task is instantaneous, costless, lossless and does not require any utilities or resources to be executed
    - The only constraints affected by the new task is the material balance and the batch size constraints
      - Adds two new continuous variables/time point to the model
    - Assignment and timing constraints are unaffected
- The mathematical formulation reduces the transfer unit and task to a one-way flow between two states



Variables associated with tasks:

$Ws_{trasfer,n}$- binary variable

$Wf_{trasfer,n}$- binary variable

$Bs_{trasfer,n}$- batch size variable

$Bp_{trasfer,n}$- batch size variable

$Bf_{trasfer,n}$- batch size variable

$Tf_{trasfer,n}$- timing variable

Mathematical formulation

- State Splitting for a continuous time STN model
  - STN model by Maravelias and Grossmann 2003

New sets

I^{trans} - imaginary transfer tasks

U^{trans} - imaginary transfer units

Reformulated mass balance constraint

$$S_{s,n} = S_{s,n-1} + \sum_{i \in I^c{}_s} Bs_{i,n} - \sum_{i \in I^p{}_s} Bf_{i,n} \quad \forall\, s, n > 1$$

Batch size constraint

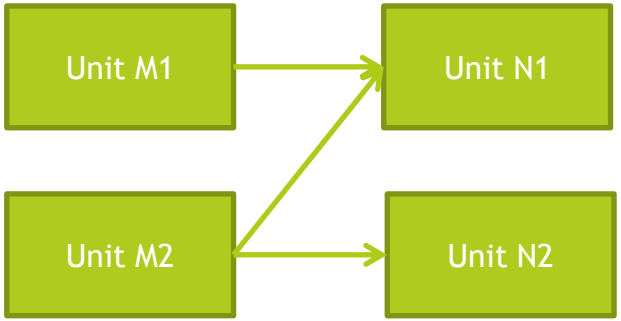
$$Bs_{i,n} = Bf_{i,n} \quad \forall\, i \in I^{trans}, n$$

Benefits of State Splitting

- State Splitting overcomes both drawback of Task Splitting as:
  - Only continuous variables are added to the model
    - The addition of only continuous variables reduces the increase in computation time compared to Task Splitting
    - The number of additional variables is less than for Task Splitting
  - Merging (and splitting) of batches is possible
    - The possibility to merge batches increases the flexibility of the mathematical model
    - In some cases this improves the solution quality
- The only drawback of State Splitting is a slight increase in model complexity

Comparison: Number of variables

Unit configuration

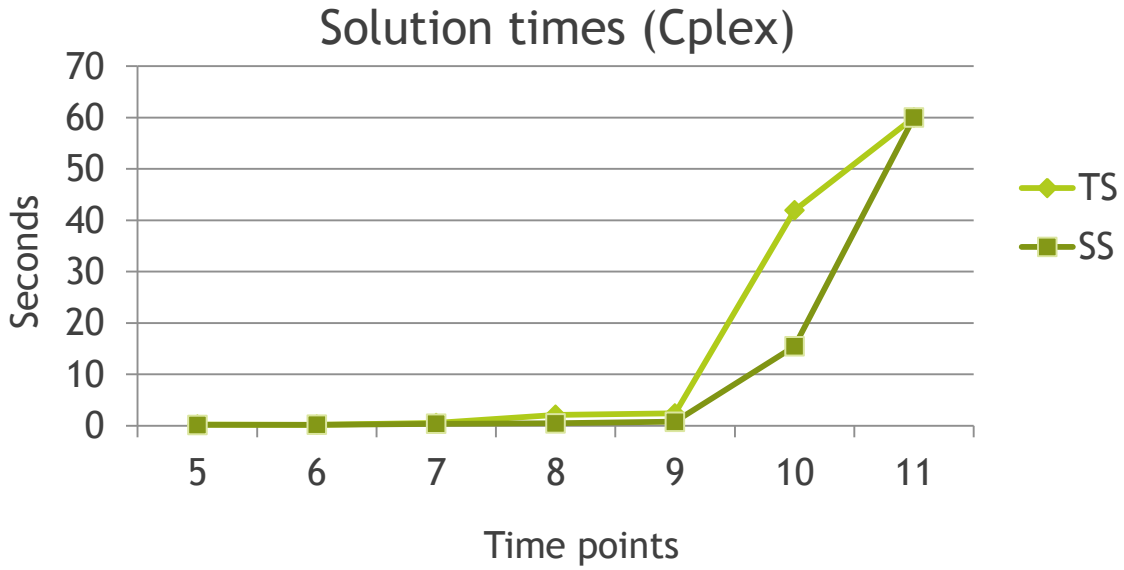


Method	# of binary *	# of cont. *
Task Splitting	10	41
State Splitting	8	37
*Number of variables / time point		

Computational results for a continuous time STN model using State Splitting

- A simple 1-commodity production planning problem with a two-stage limited connectivity unit setup
  - Objective function: maximize profit
  - Fixed time horizon (196 hours)

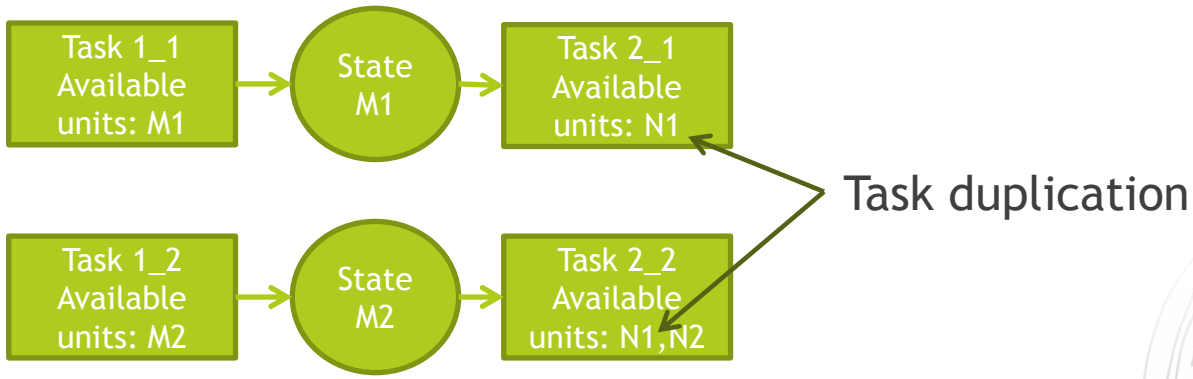
    Incremental number of time points



Improving solution quality

- State Splitting is able to produce better results than Task Splitting when:
  - The optimal solution includes at least one occurrence of batch merging
  - This is not possible in Task Splitting due to the following:
    - Tasks are duplicated
    - The allocation constraint only allows one task to be executed on each unit
  - In State Splitting this can be done because no tasks have been duplicated

Example: Task duplication

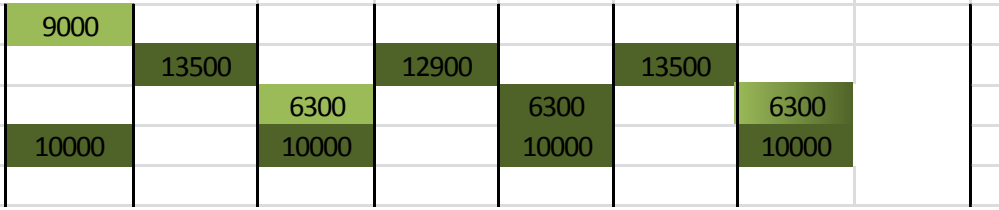


Improving solution quality

State Splitting

Objective value: 94240

Total production: 58900

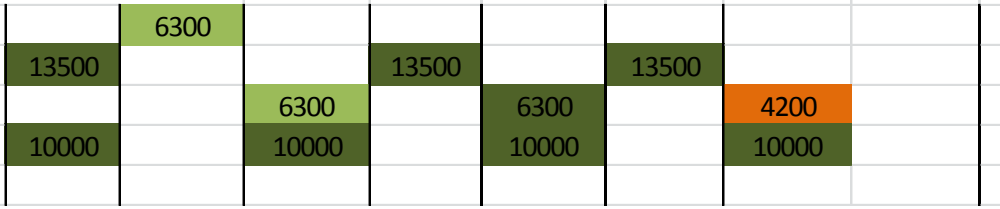


S1		9000	2700	2700	2800	2800
S2			3500	3500		
Transfer S2->S1				6400		3500

Task Splitting

Objective value: 90880

Total production: 56800



S1						
S2	10000	13500	3500	3500	700	700

Conclusions

- State Splitting produces smaller optimization problems than a corresponding Task Splitting formulation
- Solution times
  - Task Splitting vs. State Splitting
    - State Splitting produced faster results than Task Splitting
    - The results are consistent regardless of solver
- Solution quality
    - If the optimal solution includes splitting or merging a batch to/from a unit with limited equipment connectivity State Splitting will find a better solution than Task Splitting

Thank you!

Questions?